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Math-Bridge

D-3.3 First Math-Bridge service for the provision and assembly of remedial content

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Author(s)	<i>Michael Dietrich</i>



eContentplus

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¹ OJ L 79, 24.3.2005, p. 1.

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Table of Contents

1	Introduction	4
2	The Math-Bridge service	5
2.1	Math-Bridge service for students	5
2.2	Math-Bridge service for tutors	8
3	Pedagogical preparations	10
3.1	Content Selection	10
3.2	Target Competencies	10
3.3	Book Generation Scenarios	12
4	Content provided by the service.....	13
4.1	Transformation Process	13
4.1.1	Slicing.....	13
4.1.2	Structure Annotation	13
4.1.3	Metadata.....	13
4.1.4	Add Semantics.....	14
4.2	Translations.....	14
4.3	Cultural Differences	14
4.4	Content Gaps.....	17
4.5	The Gap Detection Tool.....	19
5	Summary	22
6	References	23



1 Introduction

In this deliverable we describe the first version of the Math-Bridge service that has been made available to the public¹. Besides the service installation we also provided a full featured demo version, which allows using the system without the need to register². The deliverable aims at a public audience which means, that it will not provide many technical details (it will provide references to further explanations) to keep it easily understandable.

The deliverable is divided into four sections, starting with a general description of the Math-Bridge service, including a description of its purpose and the user groups targetted. The next sections will briefly explain the necessary tasks undertaken to achieve the goals mentioned in the service description. Here we focus on the preparation steps needed from a pedagogical point of view, the transformation, translation and completion of the contents. In the final summary section we provide an outlook on the tasks that are planned to improve the service in the future.

¹ URI of the Math-Bridge service: <http://service.math-bridge.org>

² URI of the service demo installation: <http://demo.math-bridge.org>

2 The Math-Bridge service

Each year thousands of school graduates all over Europe start higher education at colleges, universities or institutes of technology. Not all of them manage to complete their studies. Besides a wrong choice of study subject this is regularly caused by a lack of mathematical competencies. This is especially true for natural sciences and engineering subjects.

To prepare students to meet the requirements of their field of study universities offer preparational courses to the students. Typically these courses last from two up to six weeks. In rare cases these courses are a full blown mathematics remedial program that can take up to 3 semesters. In these courses the teachers try to teach the students everything they need, but as this is often done using conventional class room teaching, the presented material is not adapted to the needs of individual students, but rather to groups of students. For some students this means that they have to repeat mathematical topics they are familiar with and thus lose precious time for topics they really have to learn. It might even happen that they don't learn the necessary topics at all, as they might not be covered in the course.

Also, exercises provided during the courses are often not specially selected for individual students, which could lead to exercises with too high or low difficulty. This can lead to frustration or boredom of the student.

Another important disadvantage of class room courses is the time related inflexibility. Lectures are scheduled and fix the times at which students have to be present. There is no way for the students to decide themselves when they want to learn.

2.1 *Math-Bridge service for students*

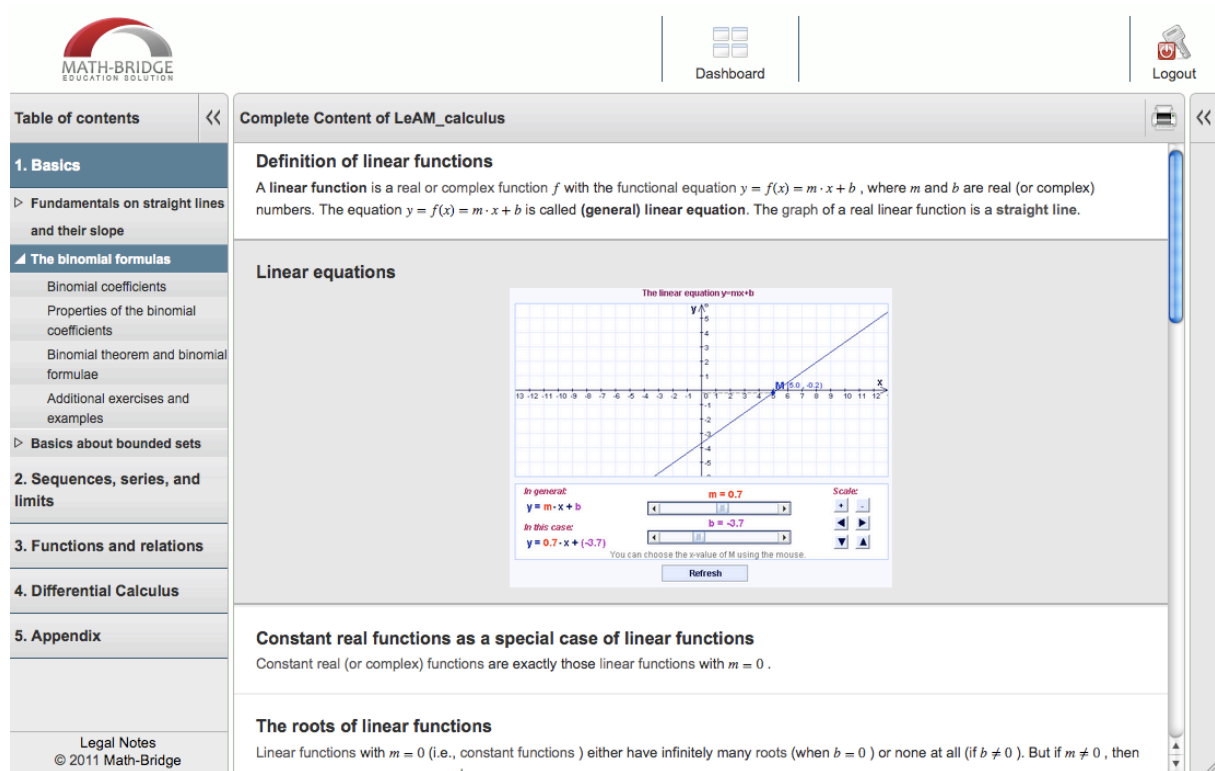
The Math-Bridge service aims to fill mathematical gaps by providing bridging courses that are individually adapted to a student's needs and knowledge.

To be able to create an individual course a student needs to register in the system and specify his intended field of study. Depending on the selected field of study the service will ask the student to solve a series of exercises. His answers are used to create an initial assessment of the student's mathematical knowledge in a so-called learner model.

The student now either uses some of the predefined bridging courses or lets the system generate one for him. Although we used the word predefined courses here, the courses are not the same for every student, as exercises and examples are adjusted to the knowledge estimation stored in his learner model.

When a student decides to use the book generation component of the service he can select from different types of books, which for example help him discover a new topic or prepare for an exam. A full list of available book types can be found in Deliverable 1.3 Pedagogical Remedial Scenarios (cf. Biehler et al., 2010b).

When working with a course the Math-Bridge service provides many functionalities which improve the student's learning experience. For easy navigation a table of contents listing all chapters, subchapters etc. of the selected course is presented to the student in the left part of the screen. To save screen space the table of contents can be hidden.



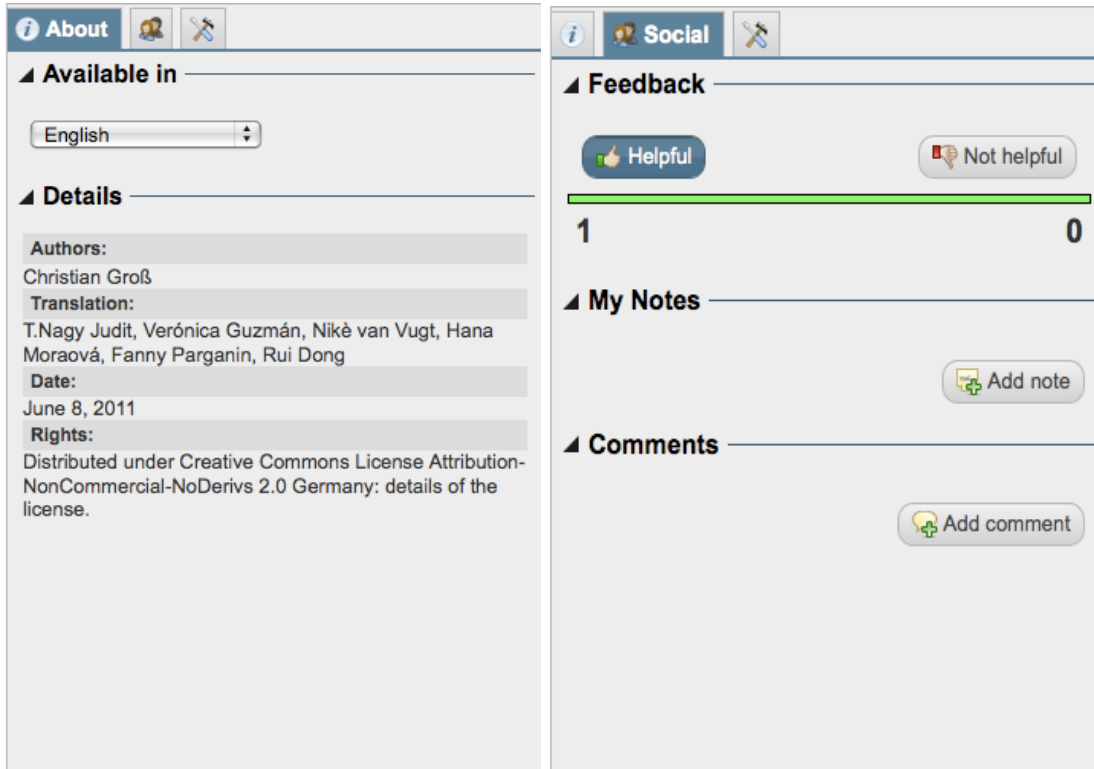
The screenshot shows the Math-Bridge service interface. On the left is a navigation menu with sections: 1. Basics, 2. Sequences, series, and limits, 3. Functions and relations, 4. Differential Calculus, and 5. Appendix. The main content area is titled 'Complete Content of LeAM_calculus' and contains the following sections:

- Definition of linear functions**: A linear function is a real or complex function f with the functional equation $y = f(x) = m \cdot x + b$, where m and b are real (or complex) numbers. The equation $y = f(x) = m \cdot x + b$ is called **(general) linear equation**. The graph of a real linear function is a **straight line**.
- Linear equations**: Includes an interactive graph titled 'The linear equation $y=mx+b$ '. The graph shows a coordinate plane with a line passing through the point $M(6.0, -0.7)$. Below the graph are input fields for $m = 0.7$ and $b = -0.7$, and a specific equation $y = 0.7 \cdot x + (-0.7)$. A 'Refresh' button is also present.
- Constant real functions as a special case of linear functions**: Constant real (or complex) functions are exactly those linear functions with $m = 0$.
- The roots of linear functions**: Linear functions with $m = 0$ (i.e., constant functions) either have infinitely many roots (when $b = 0$) or none at all (if $b \neq 0$). But if $m \neq 0$, then

At the bottom left of the page, there is a 'Legal Notes' section with the text '© 2011 Math-Bridge'.

(fig. 1 Screenshot of a course in the Math-Bridge service)

The learning material is presented in the centre of the page. The learning material is built up from smaller knowledge units (called learning objects) that each contain one chunk of mathematical knowledge. Like in other web-based application media like images, movies or applets are used to illustrate a topic or let a student explore some facts in more detail. As soon as a student selects a learning object by clicking it further information about the learning object is displayed in the panel on the right-hand side of the page. Besides information on the authorship of the learning object in question a student can create private and public comments about the learning object, read what other users say about the learning object or use social features such as like/dislike a learning object.



The screenshot displays two side-by-side panels from the Math-Bridge service interface.

About Panel:

- Available in:** A dropdown menu currently set to "English".
- Details:**
 - Authors:** Christian Groß
 - Translation:** T.Nagy Judit, Verónica Guzmán, Nikè van Vugt, Hana Moraová, Fanny Parganin, Rui Dong
 - Date:** June 8, 2011
 - Rights:** Distributed under Creative Commons License Attribution-NonCommercial-NoDerivs 2.0 Germany: details of the license.

Social Panel:

- Feedback:**
 - Buttons for "Helpful" (thumbs up) and "Not helpful" (thumbs down).
 - A progress bar showing 1 helpful and 0 not helpful votes.
- My Notes:**
 - An "Add note" button.
- Comments:**
 - An "Add comment" button.

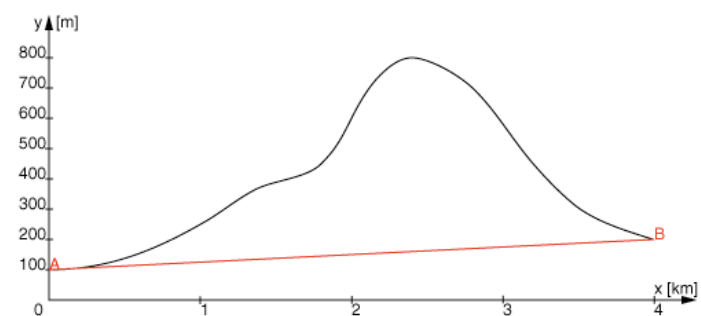
As the Math-Bridge service is intended to be used all over Europe the learning materials are currently translated to seven languages (Dutch, English, Finnish, French, German, Hungarian and Spanish). From the third panel it is possible to switch the language for a single learning object in a page. This is useful if a student wants to find the name of a mathematical concept in a language different from his own.

To foster mathematical knowledge the Math-Bridge service offers interactive exercises, which a student uses to train what he has learned. Math-Bridge offers basically two kinds of exercises.

In a multiple/single choice exercise, a student chooses the correct answer(s) from a set of choices and in fill-in-the-blank exercises where a student enters a solution in a text field. An input-editor is available to enter complicated formulæ. If a student wants to check his progress, he can take a test.

Learning Material
Exercise X

Compute the average slope of the depicted curve between $A = (x_A, y_A) = (0, 100)$ and $B = (x_B, y_B) = (4000, 200)$.



$m_{AB} =$

Evaluate
Hint
Give Up

(fig. 2 An interactive exercise)

2.2 Math-Bridge service for tutors

The Math-Bridge service wants to support tutors in teaching. A teacher can use the Math-Bridge service for many aspects of the learning process. As Math-Bridge provides a large corpus of mathematical learning objects, the time needed to find usable materials is reduced. Math-Bridge encourages the reuse of existing material by providing a tool to create new courses from existing learning material and by licensing the courses using Creative Commons licenses.

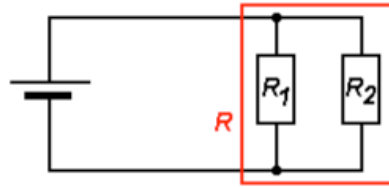
Like students, tutors can use the book generation tool to create new courses, but they are offered more options. Tutors have the possibility to generate books either for themselves using their own user model or for their students where no user model is applied at all. They can also specify the field of study (i.e. physics, engineering etc.) the book should be usable in. Setting different fields of study has an effect on exercises and examples presented in the book. The presented exercises/examples will be related to the field of study, for example a student of electrical engineering will see exercises similar to the one presented in figure 3.

To be able to present exercises or examples for every field of study the Math-Bridge service provides a large number of so-called application relevant learning objects (cf. Biehler et al., 2011).

Tutors set this options in the **Additional Options** box in the summary page of the book generation process.

parallel circuits ★★★

In parallel circuits, the multiplicative inverse of the total electrical resistance R is equal to the sum of the multiplicative inverses of the single resistances.



We regard a parallel circuit with two single resistances R_1 and R_2 :

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$$

We want to transform this equation, so that we obtain a simple formula for the total electrical resistance R . Therefore, we calculate the multiplicative inverse of the term

$\frac{1}{R_1} + \frac{1}{R_2}$, and we obtain:

$$R = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2}} = \frac{1}{\frac{R_2}{R_1 R_2} + \frac{R_1}{R_1 R_2}} = \frac{1}{\frac{R_1 + R_2}{R_1 R_2}} = \frac{R_1 R_2}{R_1 + R_2}$$



JE

(fig. 3 Exercise For Electrical Engineering Field Of Study)

This new courses can either be published to all users of the service or to a particular user group. Providing a sufficient amount of exercises to the students is a very time consuming process, as the tutor has to invent appropriate problem statements, deliver these to his students and probably correct the problems. Almost all of these tasks can be taken over by the Math-Bridge service. This is possible because we have integrated a large number of domain reasoners. Domain reasoners are computer programs which contain knowledge about a particular mathematical domain, for example derivatives. With the help of such a domain reasoner it is possible to detect errors made by students while solving the exercise, and respond to the error by giving appropriate feedback. Domain reasoners can also suggest the next possible step of a solution or, if desired generate the complete solution.

The domain reasoners have been integrated (cf. Goguadze et al., 2011) in such a way that a tutor only has to provide the kind of task the student should work on and the exercise will be automatically generated. In case the tutor wants to have more control over the exercises it is also possible to author parts of the exercise manually and only use the domain reasoner for evaluations.

As described in the previous section, sets of exercises can be grouped together to tests or exercise sheets, sent to the students and evaluated automatically.

To keep track of the student's results and learning progress a reporting functionality is integrated in the service. Of course, tutors can only create reports about the groups for which they are the assigned tutor.

Finally the Math-Bridge service can be integrated, using single sign-on, into learning management systems like Moodle, Ilias and CLIX. Single sign-on means that students only have to log-in to the LMS and are automatically logged-in to the Math-Bridge service.

3 Pedagogical preparations

To provide all components of the service mentioned in the previous section, we need to carefully prepare many aspects of the material, the interactions, and the software. From a pedagogical point of view the main components we have to specify are the competencies needed for the different fields of studies, which topics we have to cover and which test questions we should use for the assessment of the user's initial knowledge.

3.1 Content Selection

To make Math-Bridge usable all over Europe we had to determine which mathematical content has to be covered. To decide on which content to include, we let all consortium members who offer bridging courses grade all topics in a taxonomy table by importance for each of the following four user groups.

1. Engineering
2. Mathematics and science
3. Teachers
4. Rest

The importance values range from 0 (not important) – 2 (very important). After this task had been completed, we calculated the mean for each topic. As we aim at a wide-ranging usage of the Math-Bridge service we selected every topic that had a mean higher than 0.66 in at least one of the user groups. For a more detailed description see Section 2 of Deliverable 1.2 Content an Assessment tools (cf. Biehler et al., 2010a).

The tables listing the importance of mathematical topics were used to specify the necessary competencies.

3.2 Target Competencies

As described in the previous subsection we let all project partners with experience in bridging courses grade mathematical topics by importance per user group. From the results we obtained we drew conclusions on the competencies that are required from a student belonging to a particular user group.

We still had to provide a technical way to specify the necessary competencies, a competency model.

We developed a three-dimensional model that based on the PISA competency model.

The first dimension is the **mathematical competence cluster** which can be one of the following

1. **Technical competency**
This competency characterizes the technical mastery of mathematical concepts and procedures
2. **Mathematical problem solving competency**
This competency requires the understanding and solving of mathematical problems
3. **Modelling competency**
This competency is related to the modelling of real world problems.
4. **Communication and reasoning competency**
This competency reflects abilities that concern the communication, representation and explanation of mathematical content

The second dimension of the competency model are achievement levels:

1. reproduction

- all competencies are on a reproductive level;
- all procedures are routine activities (computing, standard algorithms, technical skills,...)
- reproduction of known facts and procedures
- the problem is given in a common problem representation
- familiar mathematical contexts

2. connection

- builds on the reproduction cluster
- at least one of the competencies is needed on an average level;
- defines manageable activities with several steps
- problems not based on simple and routine situations
- quasi familiar settings

3. reflection

- builds on the connection cluster
- at least one competency is needed on a high level;
- activities are complex;
- reflection and networking is needed
- planning and implementing is needed
- problem setting is close to real problems

The third dimension of the competency model describes the mathematical domain. For that purpose we created a domain ontology¹ for bridging courses for mathematics.

Again we used the importance table to create an initial version of the ontology.

This table was iteratively refined when we were annotating our content with metadata.

Further information on the competency model and the domain ontology can be found in Deliverable 1.1 Target Competencies (cf. Biehler et al., 2009)

¹ *“In the context of computer and information sciences, an ontology defines a set of representational primitives with which to model a domain of knowledge or discourse. The representational primitives are typically classes (or sets), attributes (or properties), and relationships (or relations among class members). The definitions of the representational primitives include information about their meaning and constraints on their logically consistent application.[...]”*

Due to their independence from lower level data models, ontologies are used for integrating heterogeneous databases, enabling interoperability among disparate systems, and specifying interfaces to independent, knowledge-based services.” (cf. Gruber 2008)



3.3 Book Generation Scenarios

The last large pedagogical issue that had to be addressed for the Math-Bridge service was book generation. Book generation provides courses that are adapted to the users needs. We had to decide which types of books we want to offer and what they should look like. The Math-Bridge service offers the following 6 types of automatically generated books:

1. Learn New
2. Rehearse
3. Workbook
4. Train Competency
5. Exam Simulation
6. CLO selection scenario

A detailed description of this book types can be found in Deliverable 1.3 Pedagogical Remedial Scenario. (cf. Biehler et al., 2010b)

4 Content provided by the service

To provide a service such as Math-Bridge a huge content database is required. We need to cover a large number of topics (the domain ontology contains approximately 600 entries), provide a sufficient amount of different examples and exercises for different fields of studies, provide enough exercises to the students to foster what they've learned, and so on.

To construct such a database, six partners in the Math-Bridge consortium which added content to the service. The **German Research Center For Artificial Intelligence** provided a course on fraction arithmetics, the **Universities of Kassel and Paderborn** provided a advancement of their VEMA¹ materials, the **Open Universiteit Netherlands** provided a course that recently developed for their basic mathematics bridging course, the **Saarland University** added a course on calculus, the **University of Vienna** provided contents from their online mathematics course² and finally **Tampere University of Technology** contributed two of their bridging course collections to the Math-Bridge content pool. Printed on paper the contributed materials constitute a pile of more than 2000 A4-pages.

4.1 Transformation Process

The contents were delivered in many different source formats like PDF, LaTeX or HTML, This content had to be transformed into the format needed by ActiveMath (cf. Melis et al., 2003, Melis et al., 2006b, Goguadze 2009), the technical platform of the Math-Bridge service. The applied transformation process consists of four steps:

4.1.1 Slicing

The source material provided by the partners consists of coarse grained learning units, which makes reuse hard. The Math-Bridge service provides adaptive content assembled from all available materials, so it needs access to fine-grained learning objects, which can easily be reused. To achieve this, the large learning units have to be sliced into smaller units, the so-called learning objects (LOs).

4.1.2 Structure Annotation

Learning objects can have different types in ActiveMath. For example, a learning object can be a definition, an exercise, or a proof. After slicing, we assign a type to each of the sliced learning objects. A listing of all used learning object types can be found in the supplement to Deliverable 1.1 – Metadata Cookbook (cf. Sosnovsky 2010).

4.1.3 Metadata

In order for the Math-Bridge service to automatically assemble courses tailored to the users needs, we need to provide particular information on learning objects to allow the tutorial component to discover appropriate learning objects. We use three types of metadata for this

¹ See website: <http://www.mathematik.uni-kassel.de/~vorkurs/>

² <http://www.mathe-online.at>

purpose: descriptive, pedagogic and semantic metadata.

- Descriptive metadata provides the data necessary to properly catalogue and discover a knowledge item.
- Pedagogic metadata describes pedagogic properties of knowledge items
- Semantic Metadata is used to structure the subject of study and learning material.

The learning objects of all collections have been annotated with metadata. (cf. Biehler et al., 2010c and Sosnovsky 2010 for an overview of the all metadata used in Math-Bridge and when to apply which metadata).

4.1.4 Add Semantics

Mathematical texts contain a lot of formulæ. In the source material, these formulæ are mostly represented as text or even worse, as images. ActiveMath uses a semantic representation for formulæ allowing features like formula search or copy-and-paste formulæ. So as a final step of content transformation all these formulæ were translated to the semantic representation used in ActiveMath.

After the contents had been transformed into the ActiveMath format and could be used with Math-Bridge service we performed some quality checks. This included technical, content related and pedagogic reviews. For each of this different review types we came up with criteria. To prevent duplication of work the criteria for the technical review had to be met before we let the translators do their work.

See Deliverable 2.1 Content Release for more detailed information on provided content, the transformation process and the applied quality management. (cf. Andrès et al., 2010)

4.2 Translations

As the Math-Bridge service is used all over Europe the contents have to be available in many European languages. For the initial version of the service we decided to provide contents in 7 languages: English, German, Spanish, French, Dutch, Finnish and Hungarian.

After the transformation steps and the quality review had been done we started the translation process, which consisted of the following steps:

1. *Preparation of files*

In this step all necessary technical changes to the files had been done.

2. *Translation to English*

Each content providing partner translated his content into English to make further translation possible.

3. *Quality Check*

4. *Translation to other languages*

All partners assigned for content translations were asked to do their job.

5. *Quality check*

Besides translating the content other components had to be translated, such as the user interface and the help system which is integrated in the service, and the mathematical formulæ had to be adjusted according to cultural differences.

4.3 Cultural Differences

As a European project, Math-Bridge aims at serving at least learners in Dutch, English,



French, Finnish, German, Hungarian, and Spanish, and thus must speak the mathematical language of the users. To ensure we meet this requirement we created the **notation census**¹ space, which we used to gather the differing notations of mathematics used in countries around Europe.

As a starting point for the notations, we used the OpenMath² symbols used in ActiveMath to represent formulæ. For each of these symbols we created a web page in our project wiki and asked the partners to provide the notation most commonly used in their country. To ensure that the provided notations were correct, we asked them to provide references (a page scan) to a textbook which uses this notation.

¹ Notation Census: <http://wiki.math-bridge.org/display/ntns/>

² See website: <http://www.openmath.org>

Primer label: 111,111;
 Segundo label: 111,111;
 Tercer label: 111,111;
 Abad.

gcd

✎ Added by [Abdelshafi Bekhit](#), last edited by [Abdelshafi Bekhit](#) on 10 Jan 2010

Greatest Common Divisor Notations

In mathematics, The greatest common divisor of two positive integers a and b , denoted $GCD(a, b)$.

Semantic

- [OpenMath's combinati/gcd](#)
- [MathML's gcd element](#)
- [MathWorld's Greatest common divisor](#)
- [Wikipedia's Greatest common divisor](#)

Observation: Arabic

In [Saudi math book for grad VII](#) we find in page number 75 the example shows gcd as 'ق.م.أ' for 63 and 42 in Arabic language.

ق.م.أ = ٢١

Observation: English

In [Bibliography](#) we find in page 191 this book shows the greatest common divisor in English.

$\text{gcd}(a, b) = \text{gcd}(b, r)$.

Observation: German

As found in [Bibliography](#) the German book shows the greatest common divisor in page 307.

$\text{ggT}(a, b)$.

Observation: Dutch

In [Bibliography](#) the Dutch book shows the greatest common divisor in page 81. See also another Dutch book, page 9, in [Bibliography](#).

Also used is $\text{ggd}(a, b) = a$, as seen in [Relaties en Structuren](#) (page 48).

$\text{GGD}(a, b) = a$.

Observation: Spanish

In page 271 in the Spanish book represents the $\text{mcd}(a(x), b(x)) = \text{mcd}(b(x), r_1(x)) = \dots = \text{mcd}(r_n(x), 0) = r_n(x)$ greatest common divisor as found in [Bibliography](#).

Observation: French

On page 101 of [Intro-math-discrètes](#), the notation and name of the gcd is described ([goto page](#)).

$\text{pgcd}(n, m)$ désigne le plus grand div

(fig. 4: Greatest Common Divisor Notations page from the notation census)

As soon as we gathered the input from the partners we started encoding the needed notations.

Cultural differences and the notation census are explained in more detail in Deliverable 4.1 Report about culturally different notions, notations and names (cf. Libbrecht et al., 2010).

Although Math-Bridge has access to very large database of learning materials, it might still happen that for some students or for a specific book generation scenario not all necessary learning objects are available. Finding these missing learning objects (gaps) is a very time consuming task. Therefore Math-Bridge provides a tool which helps to find these gaps.

In the following we will shortly describe the detectable types of gaps and the tool that supports finding them. The gaps found and the ways we closed them is explained in Deliverable 3.1 Report on Content Gaps Detected and Filled (cf. Miilumäki et al., 2011).

4.4 Content Gaps

In this section we describe the different kinds of content gaps that can appear in Math-Bridge collections. But before we describe these gaps we first have to briefly explain how content is encoded for Math-Bridge.

The knowledge representation used to encode learning materials in Math-Bridge is based upon OMDoc and LOM representation standards (cf. Kohlhasse 2000, IEEE 2002). Every learning item (or learning object) contains about one paragraph of text and has a type which specifies its primary function. We can divide the learning objects used in Math-Bridge into two main categories, namely fundamentals and auxiliaries. Fundamentals provide general types of mathematical knowledge like definitions or theorems. Auxiliaries contain additional information about the fundamentals in the form of examples, exercises and texts.

Every learning object contains metadata which further describes its pedagogical, organisational and semantic aspects. Exercises and examples have a difficulty level and train specific competencies. Auxiliary learning objects are always connected to fundamental learning objects using the **for** relation. To specify the connection between fundamental learning objects authors can provide pedagogical and domain-dependent relations.

```

<exercise id="exc_sets-40">
  <metadata>
    <Format>AMEL1.0</Format>
  </metadata>
  <extradata>
    <relation type="for">
      <ref xref="mbase://mb_concepts/mb_logic_and_foundations/_02_02_01_01_Set" />
      <ref xref="mbase://mb_concepts/mb_logic_and_foundations/_02_02_01_08_Subset" />
      <ref xref="mbase://mb_concepts/mb_logic_and_foundations/_02_02_01_05_In_Notin" />
    </relation>
    <competency value="technical" level="1" />
    <competency value="solving" level="2" />
    <competency value="modeling" level="0" />
    <competency value="reasoning" level="2" />
    <difficulty value="medium" />
  </extradata>
  </metadata>
  <CMP xml:lang="de">
    Selfassessment<br />
    <br />
    <highlight type="important">Aufgabenstellung:</highlight> Sei  $A = \text{set}(2, 3, 5)$  und  $B$  die Menge aller Primzahlen. Geben Sie Beziehungen zwischen diesen beiden Mengen an.
    <highlight type="noticable">Falsche Antworten:</highlight>
  </CMP>
  <CMP>
    Selfassessment<br />
    <br />
    <highlight type="important">Task:</highlight> Let  $A = \text{set}(2, 3, 5)$  and  $B$  the set of all prime numbers. Please name the relations between these two sets.
    <highlight type="noticable">Wrong answers:</highlight>
  </CMP>
  ...

```

(fig. 5: Example of an exercise learning object with *for*-relation)

This fine grained knowledge representation is used in several components of the Math-Bridge service like PAIGOS, for example, the component responsible for assembling sequences of learning objects necessary to achieve a specific learning goal based upon the prior knowledge of the current user (cf. Ullrich 2008); the build-in student model uses the relations between the learning objects to perform intelligent updating of the students' knowledge assessment while they work with the content (cf. Faulhaber et al., 2008); the search function allows

discovery and exploration of learning objects based on their metadata (cf Melis et al., 2006a).

After this short overview on knowledge representation we can have a look at the different kinds of content gaps. The gap detection tool detects three different kinds of gaps in Math-Bridge collections: structural gaps, linguistic gaps and didactic gaps.

Structural Gaps

There are three main sources of structural gaps:

1. The metadata of the learning object or its content is unrecoverably corrupted and cannot be read/restored/presented correctly by the educational system. This can be a result of a syntactic error e.g. typos or missing quotes. Such gaps are usually caught during authoring; However, sometimes these gaps slip through.
2. A learning object lacks metadata connecting it to the rest of the educational material. For example, an exercise misses the relationship to the target concepts it helps to train.
3. A learning object contains incorrect relational metadata. There can be multiple types of relations between learning objects within a content collection: prerequisite / outcome relations, links between exercises and concepts, relations specifying ordering of learning objectives. Inconsistencies in such relations are hard to find; however, detection of typical cases has been implemented (e.g. a learning object referring to itself or linking to a non-existing learning object).

Linguistic Gaps

The learning object is missing a translation to a language in which it should have been translated. The gap detection tool detects partially (content or titles) or fully (content and titles) missing translations.

There may be many reasons for missing translations, for example the learning object may have been added after the main translation effort was already finished or the translator forgot to specify the correct language metadata.

Didactic Gaps

Ensuring that existing learning objects provide correct metadata and appropriate translations is not enough for building a gap-free content collection. To be didactically complete, the collection should contain enough learning objects for all possible (or, at least, typical) learning situations. This is especially important for adaptive learning environments as they sequence learning objects dynamically based on the current state of the student model and, hence require a greater variety of learning objects in terms of their didactic properties. When an adaptive system fails to find a learning object with desired parameters, it tries to deal with the situation by finding the next best learning object. The latter option will put an extra burden on the student; s/he can receive, for example, an exercise that is too hard or too easy, or has been developed for another field of study

(and therefore uses unfamiliar terminology) or educational level (and therefore the level of detail is unfitting).

We have implemented a didactic gap detection component that relies on a practical approach to the verification of didactic completeness of learning content collections. Instead of implementing learning items for all possible situations, we first try to see which situations are possible. The didactic gap detection component employs the ActiveMath's course generation engine PAIGOS as a test bed, by providing it with generated typical user models and receiving gap reports. The rationale behind this approach is that the entire population of typical students taking the course on one hand can be characterised with a limited set of typical user profiles, and on the other is well-known to the author developing the course. Hence s/he can feed these typical user profiles to the gap detection component, which will launch the course generator in the debugging mode and intercept its requests for learning objects with non-existing characteristics. Such requests will be then presented to the author as didactic gaps.

As an example let's assume that we want to use PAIGOS to generate a book for very good students. This would for example mean that exercises integrated in the book should be difficult. So PAIGOS would send requests for learning objects with type exercise and difficulty **hard** to the database providing the mathematical content. Every time the mathematical database cannot find an appropriate learning object we have found an didactic gap.

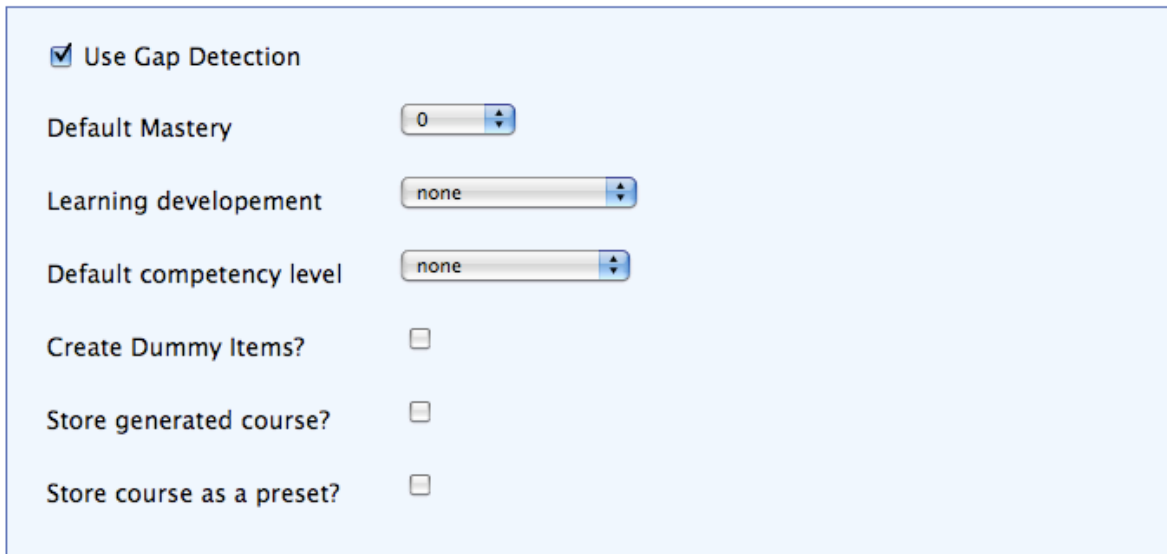
4.5 The Gap Detection Tool

The Gap Detection Tool provides support to detect all three categories of gaps described in section 4.4 of this document.

For structural gap detection the author just selects the name of the collection that should be checked for gaps and clicks on the check button. This is all information requested by the tool. For linguistic gap detection the language, that needs to be checked, has to be specified as well. To perform didactic gap detection the book generation process has to be started. This can either be done by selecting "**Create a book**" from the Math-Bridge main menu or by choosing "**Create New Preset**" from the gap detection main menu.

The author has to follow the book generation procedure which currently consists of selecting the domain of the book to be created, the desired pedagogical scenario, and refining the selected topics. The book generation process is concluded in a summary page where the author has the possibility to configure the use of the gap detection tool.

Gap Detection



Use Gap Detection

Default Mastery

Learning development

Default competency level

Create Dummy Items?

Store generated course?

Store course as a preset?

(fig. 6 Screenshot of Gap Detection configuration during course generation)

The **Default Mastery** option specifies the current knowledge of the student of the selected topic. The value ranges from 0 to 100%, which is used to check that suitable books can be generated for every student no matter how high (or low) her initial knowledge on the topic is.

Learning development specifies the learning speed of the user of the generated book, The value ranges from poor student to excellent student. This option influences the generated book in the way that better students get harder exercises earlier than less talented students. The **Competency Level** option specifies the level of competence the student has achieved already. Here the three values used in Math-Bridge (reproduction|connection|reflection) can be used. None is selected when the competency level should be ignored.

The following three checkboxes further simplify the work of the author.

If **Create Dummy Items** is checked, the gap detection tool will generate learning object skeletons for detected gaps which are prefilled with the metadata the book generator asked for and could not find. So, the author “just” has to provide the content suited for the proposed metadata. When **Store generated course** is checked the generated book is available in the authors main-menu for further work like removal/addition of learning objects or modifying its table of contents. After the tutor is satisfied with the generated book he can publish for his students (or for everyone if wanted).

The final checkbox **Store course as a preset** will add a preset to the gap detection tool main menu, which will save the author the time to go through the book generation process again if he wants to perform a didactic gap detection with the same options again.

The results of a gap detection run are presented as a list of learning objects separated by collections.



Structural Gaps for collection tutContent

Items without for relations	
show items (219)	
Items with pointers to non-existing items	
hide items (75)	
mbase://tutContent/Epaeoleellinen_raja-arvo/def23 [jE]	symb
mbase://tutContent/Epaeoleellinen_raja-arvo/def24 [jE]	symb
mbase://tutContent/Funktioiden_laskuoperaatioita/def14 [jE]	symb
mbase://tutContent/Funktioiden_laskuoperaatioita/def15 [jE]	symb

(fig. 7: Result list of structural gap detection)

5 Summary

This deliverable describes the first version of the Math-Bridge service as it is currently available on-line, including the needed pedagogical preparations, the performed transformation steps and all needed steps to ensure the completeness of the material offered. It shall be used to attract more users all over Europe, as well as interested institutions or companies, which might be interested in providing additional contents or want to run their own installations.

We are going to extend the functionalities of the service by integrating more domain reasoners, computer algebra systems and assessment systems. Also we want to extend the number of learning management systems we can integrate with.

Although our database of mathematical contents is already quite big, it might still lack some contents. Therefore we plan to transform and add more content collections provided by persons or institutions outside the Math-Bridge consortium.

6 References

Andrès, E.; Dietrich, M.: Math-Bridge: Deliverable 2.1 - Content Release. (2010)

Biehler, R.; Hochmuth, R.; Fischer, P. R.; Wassong, T. Math-Bridge: Deliverable 1.1 - Target Competencies. (2009)

Biehler, R.; Hochmuth, R.; Fischer, P. R.; Wassong, T.; Pohjolainen, S.; Dr. Nykänen, O.; Silius, K.; Miilumäki, T.; Rautiainen, E.; Mäkelä, T.: Math-Bridge: Deliverable 1.2 - Content and Assessment Tools. (2010a)

Biehler, R.; Hochmuth, R.; Fischer, P. R.; Wassong, T.: Math-Bridge: Deliverable 1.3 - Pedagogical Remedial Scenarios. (2010b)

Biehler, R.; Hochmuth, R.; Fischer, P. R.; Wassong, T.: Math-Bridge: Supplement to D 1.1 – Codebook for assigning mathematical competencies, achievement levels, domain ontology and difficulty levels. (2010c)

Biehler, R.; Hochmuth, R.; Fischer, P. R.; Wassong, T. Math-Bridge: Deliverable 3.2 – Application-relevant content added. (2011)

Faulhaber, A.; Melis, E.: An Efficient Student Model Based on Student Performance and Metadata. In: Proceedings of ECAI, pp. 276-280. IOS Press, Patras (2008)

Gogvadze, G.: Representation for Interactive Exercises. In: Proceedings of the 8th International Conference on Mathematical Knowledge Management. MKM-2009. (2009)

Gogvadze, G.; Jeuring, J.; Winterstein, S.: Math-Bridge: Deliverable 6.1 - Integration of External Tools. (2011)

Gruber, T.: Ontology. (2008) In: Liu, L and Tamer Özsu, M.; eds.: Encyclopedia of Database Systems, Springer Verlag.

IEEE: IEEE standard for learning object metadata (IEEE 1484.12.1.2002), Learning Technology Standards Committee (2002)

Kohlhase, M.: OMDoc: Towards an Internet Standard for the Administration, Distribution, and Teaching of Mathematical Knowledge. In: Proceedings of Artificial Intelligence and Symbolic Computation, pp. 32-52. Springer Berlin / Heidelberg (2000)

Libbrecht, P.; Embacher, F.: Math-Bridge: Deliverable 4.1 - Report about culturally different notions, notations and names. (2010)

Miilumäki, T., Biehler, R., Dietrich, M, Embacher, F., Fischer, R., Hochmuth, R., Jeuring, J., Kangas, J., Lodder, J., Nykänen, O, Pohjolainen, S., Sosnovsky, S, Vásárhelyi, É., Wassong, T.: Math-Bridge: Deliverable 3.1 - Content Gaps Detected and Filled (2011)

Melis, E., Büdenbender, J., Gogvadze, G., Libbrecht, P., Pollet, M., Ullrich, C.: Knowledge Representation and Management in ActiveMath. In: Proceedings of the first Conference on Mathematical Knowledge Management, MKM'01, a special issue of the Annals of Mathematics and Artificial Intelligence and in Annals of Mathematics and Artificial Intelligence, Special Issue on Management of Mathematical Knowledge. (2003)

Melis, E., Libbrecht, P.: Methods for Access and Retrieval of Mathematical Content in Activemath. In: Takayama, J. G. N., Iglesias, A. (eds.), Proceedings of ICMS-2006, LNCS, vol. 4151. Springer, Heidelberg (2006a)

Melis, E., Gogvadze, G.: Representation of Misconceptions. In: Cognition and Learning in the Digital Age, CELDA-2006. (2006b)

Sosnovsky, S. (2010): Supplement to D1.1 - Metadata Cookbook. Retrieved April, 20th 2011 from http://subversion.dfki.de/math-bridge/private/WP01_Pedagogical_Preparation/cookbook_final.pdf

Ullrich, C.: Pedagogically Founded Courseware Generation for Web-Based Learning- An HTN-Planning-Based Approach Implemented in PAIGOS. In: Lecture Notes in Artificial Intelligence (5260). Springer, Heidelberg (2008)

Vásárhelyi É., Rózsa, G. Wintsche, G.: Math-Bridge: Deliverable 5.1 – Multi lingual quality content. (2011)