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Math-Bridge

D 3.2 – Application-relevant content added

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e*Content*plus

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[1] OJ L 79, 24.3.2005, p. 1.

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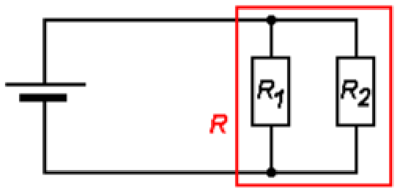
1 Introduction

This deliverable describes the process of selecting learning objects that deal with application-relevant content within the project Math-Bridge. Application-relevant LOs are examples and exercises for different fields of study. These LOs are adopted from the content of bridging courses for students at the partner universities and are needed for the specific remedial applications, which will adapt to the students' field of study and will motivate them. Many application-relevant examples and exercises/problems are needed for the targeted fields of study, e.g., for engineering, physics, chemistry, or computer science. Such specific LOs support students' motivation and show, why mathematics is relevant for their non-mathematical subjects. Since our experience shows that many students fail to solve such problems, these needs to be learned in the Math-Bridge system.

The following screenshot shows an example for an application-relevant LO (Science Cluster) which is taken from the material (http://service.math-bridge.org/ActiveMath2/search/show.cmd?id=mbase://kasselContent/Binomische_Formeln_2/rg-18). The example deals with parallel circuits where fractions are applied to calculate the total electrical resistance. The example shows that the inverse of the total electrical resistance can be calculated by the sum of the multiplicative inverse of the single resistance, which is an equation that can be memorized easily by the students. Still it does not give a direct solution to the given problem: The equation first needs to be solved for R by manipulating algebraic fractions. Thus, fractions and algebraic manipulations are prerequisites for this example.
The example is on a medium difficulty level, needs the use of mathematical techniques on a reproductive level (score 1) and mathematical problem solving on a connective level (score 2).

parallel circuits

In parallel circuits, the multiplicative inverse of the total electrical resistance R is equal to the sum of the multiplicative inverses of the single resistances.



We regard a parallel circuit with two single resistances R_1 and R_2:

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$$

We want to transform this equation, so that we obtain a simple formula for the total electrical resistance R. Therefore, we calculate the multiplicative inverse of the term $\frac{1}{R_1} + \frac{1}{R_2}$, and we obtain:

$$R = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2}} = \frac{1}{\frac{R_2}{R_1 \cdot R_2} + \frac{R_1}{R_1 \cdot R_2}} = \frac{1}{\frac{R_1+R_2}{R_1 \cdot R_2}} = \frac{R_1 \cdot R_2}{R_1+R_2}$$

Like Dislike

jE

Focusing on mathematical bridging courses, application-relevant content is moreover important for some other purposes:

- Most of the participants within mathematical bridging courses are not studying mathematics as main topic. In their respective fields of study, mathematics is mostly used as an "auxiliary science". Hence, it is important especially for first-year students to encourage use of mathematics in their specific context. For this, it is necessary to offer application-relevant content within Math-Bridge.

- Furthermore, application-relevant content helps to connect learning mathematics with the students' living environment: Hence they can see a concrete context for the usage of mathematics in their everyday life (cf. Krivsky, 2003, p. 23).
- As described above, application-relevant content will also increase the adaptivity of the Math-Bridge system to the learner's field of study. Within the learning system, this is the third criterion for the selection of learning objects next to the learner's individual achievement and the selected remedial scenario (cf. Biehler et al., 2010b).
- Next to the outer-mathematic applications, the inner-mathematic applications are also relevant for learning; following the ideas of constructivism, inner-mathematic applications can be used to develop connections between the different mathematical domains, which increase the student's understanding of the content.

In view of these purposes, we structured task 3.8 "Add application-relevant LOs", which leads to this deliverable, in the following four steps:

1. We defined five clusters grouping the fields of study.
2. Each content partner then counted the application-relevant LOs available in his own collections in view of these five clusters.
3. Afterwards, we analyzed the results of step 2 and step 3 in order to rate the quality of the existing application-relevant LOs and to identify content gaps for each of the clusters with regard to the application of the content.
4. Finally, each content partner assembled the identified application-relevant LOs into five books, according to the five clusters of fields of study.

Within this document, we will briefly describe these four steps. The collections that derive from our work are the collection of LOs (represented as books within Math-Bridge) which can be found on the common Math-Bridge service following this URL: http://service.math-bridge.org.

The books are named as follows:

- *application-relevant content: Engineering Cluster,*
- *application-relevant content: Science Cluster,*
- *application-relevant content: Mathematics Cluster,*
- *application-relevant content: Pedagogy Cluster* and
- *application-relevant content: Others Cluster*.

2 Description of the several steps to this deliverable

In this chapter we will describe the different steps of our work. The main goal was (1) to get an overview of our application-relevant learning objects and (2) to provide them as collections on a server as demonstrator.
Therefore, the first step was to cluster the fields of study in order to get a clearer overview. In the second step every content partner counted his application-relevant content. In the third step we created a book for each cluster of fields of study and in the last step we analyzed the results of counting the application-relevant learning objects.

2.1 Step 1: Clustering of fields of study

Analyzing the material provided by the content partners, we found out that LOs are usually not only usable for one specific field of study but for a cluster of fields of study instead. For instance, it is difficult to find a difference between LOs that are relevant specifically for industrial engineering, but are irrelevant for environmental engineering and other engineering fields. These clusters of fields of study are often taught together in mathematic lectures at university level and are therefore also grouped together in the bridging courses at the beginning. Hence, the learning material that is used in these courses is mostly not designed for a specific field of study, but for a cluster of fields of study instead.

Having a look at the specific material provided by the Math-Bridge's content partners one can find the same: most of the application-relevant learning objects that are available within Math-Bridge are not designed in view of a narrow field of study, their application is more orientated to a certain cluster of fields, such as *Engineering* for instance.

Therefore, we had to identify these clusters of fields of study and we had to differentiate in view of the needs of mathematical bridging courses. For this, we relied on the list of fields of study as defined for our metadata (cf. Sosnovsky, 2010).

We identified the following five clusters of fields of study for Math-Bridge: *engineering, mathematics, science, pedagogy* and *others*. The following table shows the mapping of the fields of study to these clusters.

Engineering	**Mathematics**	**Science**	**Pedagogy**	**Others**
Engineering	Computer science	Biology	Psychology	Economics
Mechanical engineering	Mathematics	Chemistry	Pedagogy	History
Industrial engineering	Statistics	Physics		Sports
Civil engineering		Geography		Architecture
Electrical engineering				Other
Environmental engineering				

A similar clustering was also used for the deliverable D1.2 on content selection (cf. Biehler et al., 2010a). The comparison still shows one important difference: for D3.2 we now split the cluster *Maths&Science* into two different clusters: *Mathematics* versus *science*. This was necessary since next to outer-mathematical applications we also needed inner-mathematical applications which are necessary to make connections between the different mathematical content domains. All learning objects that concern inner-mathematical applications belong to this cluster. As inner-mathematical application we understand in Math-Bridge the use (of aspects) of the current content domain to solve problems or to simplify some processes in other content domains. One example is the use of the binomial formulas to calculate terms like 1.01^2 or 0.97^2. Another example is Newton's method that uses the slope of a function in several points to approximately calculate the root of a function.

Another cluster that needs further specification is *pedagogy*. For this cluster we have the pre-service teachers in mind that surely will also be part of the Math-Bridge learners. Shulman (1986) described several aspects of the pedagogical content knowledge that teachers should learn: knowing "the most useful forms of representation of [the most regularly taught topics], the most powerful analogies, illustrations, examples, explanations, and demonstrations" (Shulman, 1986, p. 9) and knowing the most important conceptions and misconceptions and strategies to eliminate these misconceptions. Hence, it is necessary that Math-Bridge should offer learning opportunities to reflect the use of representations, analogies, etc. on a metacognitive level. For example within the VEMA-Material (cf. Biehler et al., in press) there are many exercises and examples that deal with the diagnosis of misconceptions in students' solutions.

2.2 Step 2: Categorizing and counting the application-relevant learning objects

In the next step the content partners DFKI, USAAR, TUT, OUNL and Kassel/Paderborn had to count the application-relevant learning objects for every cluster observed in their content. To get a clear overview of the application-relevant learning objects available and of possible gaps in the content, we reused our ontology as described in D1.1 (cf. Biehler et al., 2009) to categorize the learning objects for each cluster concerning the content domain. Using the results from D1.2 content selection (cf. Biehler et al., 2010a) we directly decided not to use the fine-grained ontology of D1.1 and to instead use a more coarse-grained version of the ontology without the distinctions on the leaf-level (level 4).

So the task for the content partners was (1) to identify all application-relevant learning objects and (2) to categorize them with regards to their cluster of fields of study and the concept in the ontology. The results of this task are attached in the appendix of this deliverable.

2.3 Step 3: Results of the former steps

Before analyzing the results of step two we made several assumptions:

1. As mentioned above, we focused on the first three concept-levels of the ontology. Analyzing the results of the step two we then decided to reduce the ontology to its second level: This was necessary to get a better overview of the possible gaps of application-relevant content. Furthermore, we saw a more important argument deriving from the structure of the content itself: the most important aspect of application-relevant content is to motivate a student for the current domain or, in case of inner-mathematical applications, for linking the domains to each other. This is not needed for every detailed concept, but rather for more broad and abstract topics. So we decided to reduce our analysis to the first two levels of the domain ontology. Thus, the most important sub- domains are covered on an adequate level of abstraction. But we made two exceptions: we added to the analysis third-level concepts *functions* and *equations*. These concepts are so important for the bridging courses that we needed to guarantee there are enough application-relevant learning objects for them.
2. We also deleted the items in section *10 Applied Mathematics* and its subsections, since the whole deliverable deals with the identification of application itself. Hence, it is useless and impossible to identify applications for "Applied Mathematics" separately.

So we took the results from step 2 and then reduced the ontology to the second level except for *function* and *equation* as described above. The results can be found in the appendix.

Analyzing the results we discovered the following:

- For *Cluster 2: Mathematics* we have an outstanding number of learning objects. Over 350 learning objects are available within the content. Having a look at the coverage of the topics, we have no application-relevant LOs only for those topics, we already identified as content gaps in deliverable 1.2 (Biehler et al., 2010a): this is mainly *geometry and topology* and *statistics and probability.* Therefore we are convinced that there should be enough learning objects to support the inner-mathematical linking between different content domains.

- Also *Cluster 4: Pedagogy* has a large number of learning objects, which mainly results from the typical mistake exercises of the content from Kassel and Paderborn. Here we also see no problems concerning gaps.
- Having a look on *Cluster 1: Engineering* we have to notice that the application-relevant learning objects are mostly available for the ontology items *calculus* and *analysis*. This is natural, as these domains of mathematics are especially needed for engineering. Most of the calculations in the context of Engineering rely on knowledge of calculus and analysis. Engineering-relevant learning objects in other sub-domains are rather not needed and sometimes can be even misleading. For this cluster, the inner-mathematical application-relevant learning objects will help to adequately cover the rest of the ontology. Thus, for Engineering, we have enough learning objects within the existing Math-Bridge material.
- For *Cluster 3: Science* we have almost the same situation as for *Cluster 1: Engineering*. The Science-relevant LOs concentrate in the topics *calculus* and *analysis*. In contrast to *Cluster 1*, there are also a few application-relevant learning objects for the topics *functions* and *equations* available. Following the argumentation concerning *Cluster 1: Engineering* we also have enough application-relevant learning objects for cluster 3.
- The results for *Cluster 5: Others* are very different: Here, most application-relevant learning objects can be found in the first chapters of the ontology: *arithmetic*, *set theory*, *functions* and *equations*. These are predominantly the content domains that are the basis for other domains. Most of these learning objects are orientated at the living environment of the students. In addition to the inner-mathematical connection, these learning objects motivate the corresponding content domains by connecting them to the students' living environment as claimed for the teaching of mathematics in school for several times. So there are also enough application-relevant learning objects available for *Cluster 5: Others*.

2.4 Step 4: Collecting the application-relevant content in books

The last step was to collect all application-relevant learning objects in separate books, one book for each cluster of fields of study. These books have benefits for lecturers as well as for students:

- Lecturers can use these books to inform themselves about application-relevant content, to get new ideas for their lectures and to enrich their books with application-relevant learning objects. For this the structure of the application-relevant-LO books will help them to find the fitting learning objects.
- Students can use these books to inform themselves about the relevance of mathematics for their field of study. Within these books, they can find the relevant aspects easily in one place.

The collections can be found on the Math-Bridge service http://service.math-bridge.org. The names of the books are *application-relevant content: Engineering Cluster*, *application-relevant content: Science Cluster*, *application-relevant content: Mathematics Cluster*, *application-relevant content: Pedagogy Cluster* and *application-relevant content: Others Cluster*.

3 Conclusion

This deliverable describes the tasks for identifying and providing application-relevant learning objects. In the first step, different fields of study were combined into five larger clusters: *Engineering*, *Mathematics*, *Science*, *Pedagogy*, and *Others*. In the second step, we categorized and counted the application-relevant learning objects with respect to the five clusters of fields of study and the Math-Bridge domain ontology. The results of this process were evaluated in the third step. In the last step the learning objects were combined into books, one for each cluster. These books are aimed at supporting lecturers in the process of enriching their material with application-relevant content, as well as helping students to learn about the relevance of mathematics to their major field of study.

The results of this work were satisfactory. We started the analysis with the inner-mathematical application which we identified as basis for connecting the content domains among each other. For this, a large number of application-relevant learning objects are available. In addition, there exist multiple learning objects for linking the basic content domains with the living environment of the students. Concerning the *Engineering* and *Science* clusters, the application-relevant learning objects concentrate in the content domains *calculus* and *analysis*. For other content domains, it is not only difficult to find fitting learning objects relevant to these fields of study, it is even senseless. Therefore, it is not necessary to develop new learning objects in order to fill these gaps. Instead, the gaps can be closed by inner-mathematical application-relevant content. This inner-mathematical application-relevant content is needed by the Engineering and Science students to realize the necessity of the basic content domains for *calculus* and *analysis*.

In summary, this deliverable shows that Math-Bridge service provides a wide-spread set of application-relevant mathematical content to support meaningful learning on the level of school to university transition.

The next step in the Math-Bridge project will be for every content partner to revise their existing books in the Math-Bridge service based on their content and to enrich them with the collected application-relevant learning objects. Here, every partner who will provide bridging courses and take part in the large-scale evaluation will use the created books to enrich their predefined books.

4 References

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5 Appendix

5.1 Results of counting the application-relevant content in step 2

		Cluster 1: Engineering	Cluster 2: Mathematics	Cluster 3: Sciences	Cluster 4: Pedagogy	Cluster 5: Others
1.0	**Numbers and Computation**	0	0	0	0	0
1.1	**Number Concepts**	0	0	0	0	2
1.1.1	Natural	0	0	0	0	0
1.1.2	Integers	0	0	0	0	0
1.1.3	Rational	0	0	0	0	2
1.1.4	Irrational	0	0	0	0	0
1.1.5	Algebraic	0	0	0	0	0
1.1.6	Real	0	0	0	0	0
1.1.7	Complex	0	0	0	0	0
1.1.8	Famous Numbers	0	0	0	0	0
1.2	**Arithmetic**	1	10	1	21	5
1.2.1	Operations	0	6	1	7	11
1.2.2	Fractions	0	71	0	6	0
1.2.3	Decimals	0	0	0	0	8
1.2.4	Comparison of numbers	0	0	0	0	0
1.2.5	Exponents	0	23	2	17	9
1.3	**Patterns and Sequences**	0	0	0	0	0
1.3.1	Number Patterns	0	0	2	2	5
1.3.2	Fibonacci Sequence	0	2	0	0	0
1.3.3	Arithmetic Sequence	0	0	0	0	0
1.3.4	Geometric Sequence	0	8	1	4	2
1.4	**Measurement**	0	0	0	0	2
1.4.1	Units of Measurement	0	0	0	0	0
1.4.2	Linear Measure	0	0	0	0	0
1.4.3	Area	0	0	0	0	0

		Cluster 1: Engineering	Cluster 2: Mathematics	Cluster 3: Sciences	Cluster 4: Pedagogy	Cluster 5: Others
1.4.4	Volume	0	0	0	0	0
1.4.5	Weight and Mass	0	0	0	0	0
1.4.6	Temperature	0	0	0	0	0
1.4.7	Time	0	0	0	0	0
1.4.8	Speed	0	0	0	0	0
1.4.9	Money	0	0	0	0	0
1.4.10	Scale	0	0	0	0	0
2.0	**Logic and Foundations**	0	0	0	0	0
2.1	**Logic**	0	6	1	4	2
2.1.1	Venn Diagrams	0	1	0	0	0
2.1.2	Propositional and Predicate Logic	0	5	0	3	2
2.1.3	Methods of Proof	0	4	0	3	0
2.2	**Set Theory**	0	0	0	0	0
2.2.1	Sets and Set Operations	0	16	0	12	4
2.2.2	Relations and Functions	0	12	3	0	17
2.2.3	Cardinality	0	2	0	0	1
3.0	**Algebra and Number Theory**	0	0	0	0	0
3.1	**Algebra**	0	11	1	0	0
3.1.1	Graphing Techniques	0	1	0	0	0
3.1.2	Algebraic Manipulation	1	25	0	14	3
3.1.3	Functions	0	40	14	31	11
3.1.4	Equations	1	18	8	15	10
3.1.5	Inequalities	1	11	0	8	1
3.1.6	Matrices	0	0	0	0	0
3.1.7	Sequences and Series	0	0	0	0	0
3.1.8	Algebraic Proof	0	0	0	0	0
3.2	**Linear Algebra**	0	0	0	0	0
3.2.1	Systems of Linear Equations	2	0	0	0	0
3.2.2	Matrix algebra	0	1	0	0	2
3.2.3	Vectors in R3	0	0	0	0	1
3.2.4	Vector Spaces	0	0	0	0	0
3.2.5	Linear Transformations	0	0	0	0	0

		Cluster 1: Engineering	Cluster 2: Mathematics	Cluster 3: Sciences	Cluster 4: Pedagogy	Cluster 5: Others
3.2.6	Eigenvalues and Eigenvectors	0	0	0	0	0
3.2.7	Inner Product Spaces	0	0	0	0	0
3.4	**Number Theory**	0	0	0	0	0
3.4.1	Integers	0	14	0	0	0
3.4.2	Primes	0	0	0	0	0
4.0	**Discrete Mathematics**	0	0	0	0	0
4.2	**Combinatorics**	0	5	0	5	1
4.2.1	Combinations	0	0	0	0	0
4.2.2	Permutations	0	0	0	0	0
4.5	**Recursion**	0	0	0	0	0
4.7	**Linear Programming**	0	0	0	0	0
5.0	**Geometry and Topology**	0	0	0	0	0
5.1	**Geometric Proof**	0	0	0	0	0
5.2	**Plane Geometry**	0	0	0	0	0
5.2.1	Measurement	0	0	0	0	0
5.2.2	Lines and Planes	0	0	0	0	0
5.2.3	Angles	0	0	0	0	0
5.2.4	Triangles	0	0	0	0	0
5.2.5	Polygons	0	0	0	0	0
5.2.6	Circles	0	1	0	0	0
5.2.7	Patterns	0	0	0	0	0
5.2.8	Transformations	0	0	0	0	0
5.3	**Solid Geometry**	0	0	0	0	0
5.3.1	Dihedral Angles	0	0	0	0	0
5.3.2	Spheres	0	0	0	0	0
5.3.3	Cones	0	0	0	0	0
5.3.4	Cylinders	0	0	0	0	0
5.3.5	Pyramids	0	0	0	0	0
5.3.6	Prisms	0	0	0	0	0
5.3.7	Polyhedra	0	0	0	0	0
5.4	**Analytic Geometry**	0	0	0	0	0

		Cluster 1: Engineering	Cluster 2: Mathematics	Cluster 3: Sciences	Cluster 4: Pedagogy	Cluster 5: Others
5.4.1	Cartesian Coordinates	0	0	0	0	0
5.4.2	Lines	0	0	0	0	0
5.4.3	Circles	0	0	0	0	0
5.4.4	Planes	0	0	0	0	0
5.4.6	Polar Coordinates	0	0	0	0	0
5.5	**Projective Geometry**	0	0	0	0	0
5.9	**Trigonometry**	0	1	0	0	0
5.9.1	Angles	0	0	0	0	0
5.9.2	Trigonometric Functions	0	1	0	0	0
5.9.3	Inverse Trigonometric Functions	0	0	0	0	0
5.9.4	Trigonometric Identities	0	0	0	0	0
5.9.5	Trigonometric Equations	0	0	0	0	0
5.9.6	Roots of Unity	0	0	0	0	0
5.9.7	Spherical Trigonometry	0	0	0	0	0
6.0	**Calculus**	4	5	14	0	2
6.1	**Single Variable**	0	0	0	0	0
6.1.1	Functions	1	1	4	0	1
6.1.2	Limits	2	9	2	6	0
6.1.3	Continuity	2	3	8	2	5
6.1.4	Differentiation	1	12	2	5	0
6.1.5	Integration	5	22	5	13	1
6.1.6	Series	1	6	1	4	0
6.2	**Several Variables**	0	0	0	0	0
6.2.1	Functions of Several Variables	0	0	0	0	0
6.2.2	Limits	0	0	0	0	0
6.2.3	Continuity	0	0	0	0	0
6.2.4	Partial Derivatives	0	0	0	0	0
6.2.5	Multiple integrals	0	0	0	0	0
7.0	**Analysis**	0	0	0	0	0
7.3	**Numerical Analysis**	0	0	0	0	0
7.3.2	Solutions of Equations	0	0	0	0	0
7.3.3	Solutions of Systems	0	0	0	0	0

		Cluster 1: Engineering	Cluster 2: Mathematics	Cluster 3: Sciences	Cluster 4: Pedagogy	Cluster 5: Others
9.0	**Statistics and Probability**	0	0	0	0	0
9.1.	**Data Collection**	0	0	0	0	0
9.1.1.	Experimental Design	0	0	0	0	0
9.1.2.	Sampling and Surveys	0	0	0	0	0
9.1.3.	Data and Measurement Issues	0	0	0	0	0
9.2.	**Data Summary and Presentation**	0	0	0	0	0
9.2.1.	Summary Statistics	0	0	0	0	0
9.2.2.	Data Representation	0	0	0	0	0
9.3.	**Statistical Inference and Techniques**	0	0	0	0	0
9.3.1.	Sampling Distributions	0	0	0	0	0
9.3.2.	Regression and Correlation	0	0	0	0	0
9.3.3.	Confidence Intervals	0	0	0	0	0
9.3.4.	Hypothesis Tests	0	0	0	0	0
9.4.	**Probability**	0	0	0	0	0
9.4.1.	Elementary Probability	0	0	0	0	0
9.4.2.	Univariate Distributions	0	0	0	0	0
9.4.3.	Limit Theorems	0	0	0	0	0
9.4.7.	Simulation	0	0	0	0	0
10.0	**Applied Mathematics**	0	0	0	0	1
10.3	**Mathematical Biology**	0	0	1	0	0
10.5	**Engineering Mathematics**	0	0	0	0	0
10.8	**Mathematics for Computer Science**	0	0	0	0	0
	Sum	22	353	71	182	111

5.2 Results of the analysis in step 3

		Cluster 1: Engineering	Cluster 2: Mathematics	Cluster 3: Sciences	Cluster 4: Pedagogy	Cluster 5: Others
1.0	**Numbers and Computation**	0	0	0	0	0
1.1	**Number Concepts**	0	0	0	0	4
1.2	**Arithmetic**	1	110	4	51	33
1.3	**Patterns and Sequences**	0	10	3	6	7
1.4	**Measurement**	0	0	0	0	2
2.0	**Logic and Foundations**	0	0	0	0	0
2.1	**Logic**	0	16	1	10	4
2.2	**Set Theory**	0	30	3	12	22
3.0	**Algebra and Number Theory**	0	0	0	0	0
3.1	**Algebra**	2	48	1	22	4
3.1.3	Functions	0	40	14	31	11
3.1.4	Equations	1	18	8	15	10
3.2	**Linear Algebra**	2	1	0	0	3
3.4	**Number Theory**	0	14	0	0	0
4.0	**Discrete Mathematics**	0	0	0	0	0
4.2	**Combinatorics**	0	5	0	5	1
4.5	**Recursion**	0	0	0	0	0
4.7	**Linear Programming**	0	0	0	0	0
5.0	**Geometry and Topology**	0	0	0	0	0
5.1	**Geometric Proof**	0	0	0	0	0
5.2	**Plane Geometry**	0	1	0	0	0
5.3	**Solid Geometry**	0	0	0	0	0
5.4	**Analytic Geometry**	0	0	0	0	0
5.5	**Projective Geometry**	0	0	0	0	0
5.9	**Trigonometry**	0	2	0	0	0
6.0	**Calculus**	4	5	14	0	2

		Cluster 1: Engineering	Cluster 2: Mathematics	Cluster 3: Sciences	Cluster 4: Pedagogy	Cluster 5: Others
6.1	**Single Variable**	12	53	22	30	7
6.2	**Several Variables**	0	0	0	0	0
7.0	**Analysis**	0	0	0	0	0
7.3	**Numerical Analysis**	0	0	0	0	0
9.0	**Statistics and Probability**	0	0	0	0	0
9.1.	**Data Collection**	0	0	0	0	0
9.2.	**Data Summary and Presentation**	0	0	0	0	0
9.3.	**Statistical Inference and Techniques**	0	0	0	0	0
9.4.	**Probability**	0	0	0	0	0
~~**10.0**~~	~~**Applied Mathematics**~~	~~0~~	~~0~~	~~0~~	~~0~~	~~1~~
~~**10.3**~~	~~**Mathematical Biology**~~	~~0~~	~~0~~	~~1~~	~~0~~	~~0~~
~~**10.5**~~	~~**Engineering Mathematics**~~	~~0~~	~~0~~	~~0~~	~~0~~	~~0~~
~~**10.8**~~	~~**Mathematics for Computer Science**~~	~~0~~	~~0~~	~~0~~	~~0~~	~~0~~
	Sum	22	353	71	182	111